4.2 Creating Pattern Rules from Models

You will need
- coloured square tiles
- toothpicks
- coloured pencils

GOAL
Use algebraic expressions to describe patterns.

Learn about the Math

Chad, Benjamin, and Toma used coloured tiles to create three different models that represent the same pattern. Each student then used the **variable** t to write a different **algebraic expression** to describe the number of tiles in each figure in the pattern.

variable
a letter or symbol, such as a, b, or n, that represents a number

algebraic expression
a combination of one or more variables; it may include numbers and operation signs

Chad's model shows $3t + 3$.

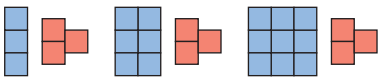


Figure 1 Figure 2 Figure 3

Benjamin's model shows $t + t + t + 3$.

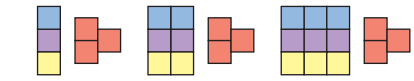


Figure 1 Figure 2 Figure 3

Toma's model shows $3(t + 1)$.

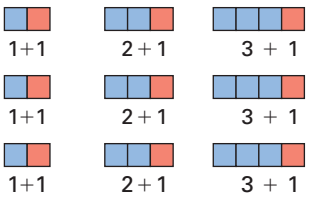


Figure 1 Figure 2 Figure 3

? How do the students' models relate to their algebraic pattern rules?

A. How did Chad use colour to show which part of his pattern stays the same and which part changes?

B. How does Chad's model relate to his algebraic pattern rule?

C. Repeat steps A and B for Benjamin's and Toma's models and algebraic expressions.

D. The total number of tiles in each figure of the pattern is a multiple of 3. Which pattern rule shows this most clearly?

Reflecting

1. How could you use coloured tiles to show that the pattern rule $6 + 3 \times (n - 1)$ also represents the same pattern?

2. How do you know that all these pattern rules must be equivalent?

Work with the Math

Example: Identifying and describing a pattern rule

Write a pattern rule to describe the number of toothpicks in each figure of this toothpick fence.

Figure 1 Figure 2 Figure 3

Hoshi's Solution

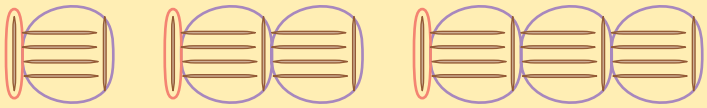


Figure 1 Figure 2 Figure 3

I used a purple pencil to circle the part of the pattern that changes. I see that each new section of fence has 5 more toothpicks.

I used a red pencil to circle the part of the pattern that stays constant. There is always 1 post at the beginning of each figure.

I wanted the number of sections in each figure to be the same as the figure number.

My variable is n. It represents the number of sections of fence. My pattern rule is $5n + 1$.

Check:
In figure 1, the total is $5(1) + 1 = 6$. ✓
In figure 2, the total is $5(2) + 1 = 11$. ✓
In figure 3, the total is $5(3) + 1 = 16$. ✓

I checked to make sure that my pattern rule works.

Tran's Solution

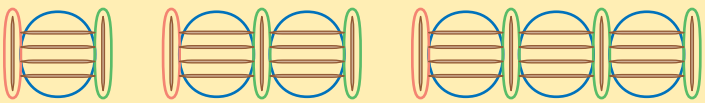


Figure 1 Figure 2 Figure 3

I circled the first post in red because that's what each figure begins with. It doesn't change.

The rest of each figure is the part that changes. Each section of fence that is added has 4 rails and 1 post.

I used the variable s to represent the number of sections of fence after the first post.

My pattern rule is $s(4 + 1) + 1$.

Check:
In figure 1, I get $1(4 + 1) + 1 = 6$. ✓
In figure 2, I get $2(4 + 1) + 1 = 11$. ✓
In figure 3, I get $3(4 + 1) + 1 = 16$. ✓

My pattern rule describes the relationship between the figure number and the total number of toothpicks.

I checked to make sure that my pattern rule works.

A Checking

3. a) Which part of this pattern changes? Which part stays the same?

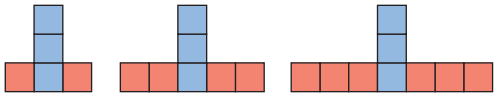


Figure 1 Figure 2 Figure 3

b) Use an algebraic expression to describe the number of tiles in terms of the figure number.

4. Describe this pattern rule in words. Then write two different algebraic expressions to describe the pattern rule.

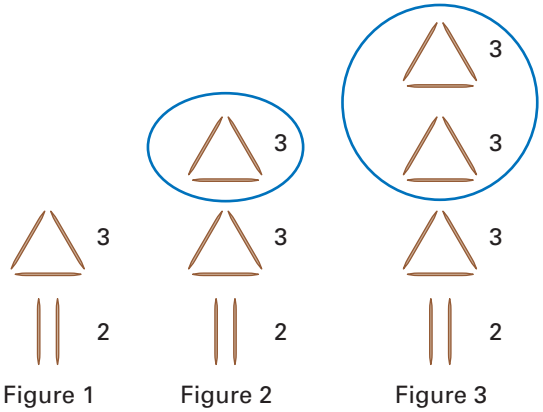
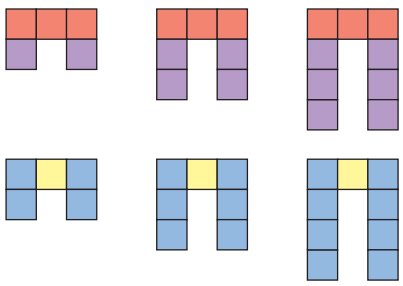


Figure 1 Figure 2 Figure 3

B Practising

5. These two models show the same pattern in two different ways.

Figure 1 Figure 2 Figure 3



a) Describe what changes and what stays the same in each model.

b) Write an algebraic pattern rule for each model.

6. The variable n represents the figure number in the following algebraic pattern rules:

A. $4n$ **B.** $2n + 2$ **C.** $3n + 1$ **D.** $4n - 1$

Which of these pattern rules describes the toothpick pattern shown below? Explain how you know.

Figure 1 Figure 2 Figure 3 Figure 4

7. Write two algebraic pattern rules for this toothpick pattern.

Figure 1 Figure 2 Figure 3 Figure 4

8. The following fence pattern starts with a gate and increases by one section in each consecutive figure. Write two algebraic pattern rules for this pattern.

Figure 1 Figure 2 Figure 3

9. a) Write an algebraic expression that describes the pattern rule for the number of red tiles.

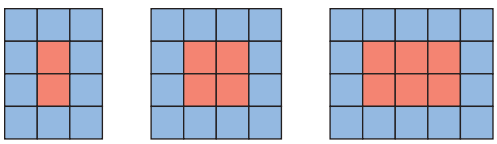


Figure 1 Figure 2 Figure 3

b) Repeat part (a) for the blue tiles. Explain your thinking.

10. Use the pattern in question 9.

a) Can you make a figure with an odd number of blue tiles? Explain.

b) How many blue tiles would you need to make a figure that has 10 red tiles?

11. Describe this pattern rule in words, and then with an algebraic expression.

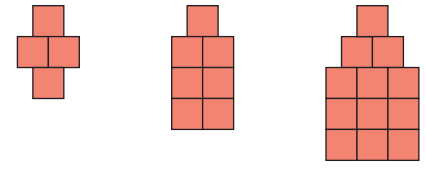


Figure 1 Figure 2 Figure 3

12. Use each description to draw a diagram and write an algebraic pattern rule.

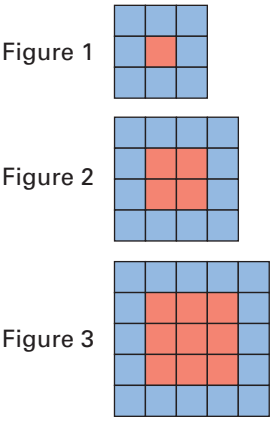
a) In figure 1, there are four tiles in a row and two tiles on top. Another row of four tiles is added on the bottom of each consecutive figure.

b) In figure 1, there are four tiles in a row and two tiles on top. Another two tiles are added on the top of each consecutive figure.

c) The total number of tiles is the figure number squared, plus the figure number.

C Extending

13. a) Write an algebraic expression that describes the pattern rule for the number of red tiles.



b) Repeat part (a) for the blue tiles.

c) Write two different algebraic expressions that describe the total number of tiles in the pattern.

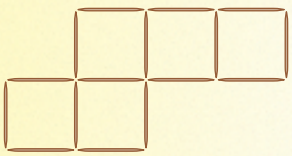
Mental Imagery

MOVING TOOTHPICKS

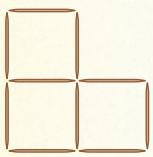
You will need
- toothpicks

Maria's little brother made the following pens for his tiny toy animals, using toothpicks.

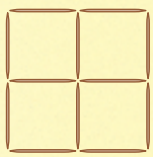
1. How can you move only two toothpicks to leave four square pens?



2. How can you move only two toothpicks to turn the L shape upside down?



3. How can you remove only two toothpicks to leave two square pens?



4. How can you remove only three toothpicks to leave three triangular pens?